$q = AK^{\alpha}L^{\beta}$ For Linearly homogeneous

① Degree of homogenuity $(\alpha+\beta)$ $q = AL^{1-\alpha}K^{\alpha}$

② $MP_L = \beta \cdot AP_L = \beta\frac{q}{L}$, $MP_K = \alpha AP_K = \alpha\frac{q}{K}$ MRTS $= \frac{\beta}{\alpha}\cdot\frac{K}{L}$.

as MP_L & MP_K are +ve $\alpha, \beta > 0$ $f_{LL} = \beta(\beta-1)AK^{\alpha}L^{\beta-2}$, $f_{KK} = \alpha(\alpha-1)AK^{\alpha-2}L^{\beta}$

To satisfy Diminish MPs $0 < \alpha, \beta < 1$

③ Prove that for ~~Linearly~~ CRS C-D. Funⁿ MPs are homogeneous of degree zero.

④ Prove that for CRS C-D funⁿ APs and MPs are funⁿ of R only

⑤ Expansion paths are straight line. $\frac{\beta}{\alpha}\frac{K}{L} = \frac{w}{r}$

passing through origin. or $\beta r K - \alpha w L = 0$.

⑥ C-D. prodⁿ funⁿ satisfies Euler's theorem.

$K\,MP_K + L\,MP_L = (\alpha+\beta)q$. [$Kf_K + Lf_L = q$ For CRS, Total output will be exhausted according to MPP theory]

For IRS Total payment would exceed output

For DRS $(\alpha+\beta<1)$ " " " less than output

⑦ If each input is paid by the amount of its MP, the relative share of total product accruing to K is $\frac{K.MP_K}{q} = \alpha$, $\frac{L\,MP_L}{q} = \beta$

Thus the exponent of each input variable indicates the relative share of that input in the total product. $(\omega_K + \omega_L = \alpha+\beta)$

⑧ Sum of output elasticies for K & L equals degree of homogenity

⑨ Each exponent of each input can be expressed as partial elasticity of output with respect to that input as

$$\alpha = \frac{MP_K}{AP_K} = \omega_K, \quad \beta = \frac{MP_L}{AP_L} = \omega_L$$

⑩ Factor intensity $= \beta/\alpha$. The higher the ratio the more labor intensive the Technique

⑪ Efficiency of Production $= A$.

⑫ Elasticity of substitution $= \dfrac{d(K/L)/(K/L)}{d(MRTS)/MRTS} = \dfrac{d(K/L)/(K/L)}{d\left(\frac{\beta}{\alpha}\cdot\frac{K}{L}\right)/\left(\frac{\beta}{\alpha}\cdot\frac{K}{L}\right)} = 1$

(Constant elasticity of substitution)

⑬ Although the funⁿ is nonlinear it can be transformed into a linear funⁿ

$\log q = \log A + \alpha\log K + \beta\log L$ or $q' = A' + \alpha K' + \beta L'$

⑭ Both inputs are required to produce as $f(K,0) = f(0,L) = 0$

⑮ Show that CRS is not irrelevant with diminishing MPs.

$q = AL^{1-\alpha}K^{\alpha}$ ~~$MP_K = \alpha\frac{q}{K} = \alpha AL^{1-\alpha}K^{\alpha-1}$ $MP_L = (1-\alpha)\frac{q}{L}$~~ $0<\alpha<1$

~~$MP_{KK} = \alpha(\alpha-1)AL^{1-\alpha}K^{\alpha-2}$ MP_L~~

$f_L = MP_L = (1-\alpha) A L^{-\alpha} K^{\alpha} > 0$ MPs are positive but diminishing $1 > \alpha,\ 1-\alpha > 0$

$f_K = MP_K = \alpha A K^{\alpha-1} L^{1-\alpha} > 0$

$MP_{LL} = -(1-\alpha)\alpha A L^{-(\alpha+1)} K^{\alpha} < 0$

$MP_{KK} = -\alpha(1-\alpha) A K^{\alpha-2} L^{1-\alpha} < 0$

(16) C-D prodn funn is strictly quasiconcave for positive K & L.

For CRS →

$f_{KL} = \alpha A K^{\alpha-1}(1-\alpha) L^{-\alpha}$

Evaluating Hessian determinant of prodn fun.

$$\begin{vmatrix} f_{LL} & -f_{LK} \\ -f_{LK} & f_{KK} \end{vmatrix} = f_{LL} f_{KK} - f_{LK}^2$$

$$= \alpha^2 (1-\alpha)^2 A^2 K^{2(\alpha-1)} L^{-2\alpha} - \alpha^2 (1-\alpha)^2 A^2 K^{2(\alpha-1)} L^{-2\alpha}$$

$$= 0$$

Therefore C-D prodn funn of degree one is concave but has linear subregions in which it is not strictly concave.

$q = A K^{\alpha} L^{\beta}$ $MP_K = A \alpha K^{\alpha-1} L^{\beta}$ $MP_L = A \beta K^{\alpha} L^{\beta-1}$

$MP_{KL} = A \alpha \beta K^{\alpha-1} L^{\beta-1}$ $MP_{KK} = A \alpha(\alpha-1) K^{\alpha-2} L^{\beta}$, $MP_{LL} = A \beta(\beta-1) K^{\alpha} L^{\beta-2}$

$$\therefore f_{LL} f_{KK} - f_{LK}^2 = A^2 \alpha\beta(\alpha-1)(\beta-1) K^{2(\alpha-1)} L^{2(\beta-1)} - A^2 \alpha^2 \beta^2 K^{2(\alpha-1)} L^{2(\beta-1)}$$

$$= A^2 K^{2(\alpha-1)} L^{2(\beta-1)} \left[\alpha\beta(\alpha-1)(\beta-1) - \alpha^2\beta^2\right]$$

$$= (1-\alpha-\beta) \frac{\alpha\beta q^2}{K^2 L^2}$$

As ~~if $\alpha+\beta>1$~~

For concavity $\alpha\beta(\alpha-1)(\beta-1) - \alpha^2\beta^2 > 0$

$(\alpha-1)(\beta-1) - \alpha\beta > 0$

$\alpha\beta - \beta - \alpha + 1 - \alpha\beta > 0$ or $-(\alpha+\beta) + 1 > 0 \Rightarrow \alpha+\beta < 1$

∴ In CRS ⇒ Concave with linear subregion   (H & Q 72)

DRS ⇒ Strictly concave

IRS ⇒ ~~Strictly Convex~~ it is negative & The prodn funn is neither concave nor convex

(17) Convexity of isoquant

$q^0 = A K^{\alpha} L^{\beta}$ or $K = \left(\frac{q^0}{A}\right)^{1/\alpha} L^{-\beta/\alpha}$

$$\frac{dK}{dL} = -\left(\frac{q^0}{A}\right)^{1/\alpha} \frac{\beta}{\alpha} L^{-\frac{(\alpha+\beta)}{\alpha}} \; ; \; \frac{d^2K}{dL^2} = \frac{\beta}{\alpha} \cdot \frac{\alpha+\beta}{\alpha} \left(\frac{q_0}{A}\right)^{1/\alpha} L^{-\left(\frac{2\alpha+\beta}{\alpha}\right)} > 0$$

∴ The isoquants will be of the desired shape for any positive values of α & β